Accelerating Proximal Gradient Descent via Silver Stepsizes

This notebook verifies the algebraic identities in Appendix B of the paper
"Accelerating Proximal Gradient Descent via Silver Stepsizes"
by Jinho Bok and Jason M. Altschuler [arXiv:2412.05497]

Definition

In[ ]:=
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(* Silver ratio *)
rho = 1 + Sqrt[2];
```

Verification of identity: certificate of (g, s) (Proposition B.9)

In[ ]:=
```
(* Left hand side of identity in Proposition B.9 *)
LHSgs = (G1) (S2 - S1) +
    rho (G3) (2 S1 + (rho^(k - 1) + 1) S3 + S4) +
    (G1 + (rho^(k - 1) + 1) G2 + G3) ((rho - 1 - 1 / rho^k) S4 + (rho + 1) S5) +
    (rho^k) (G2) (S1 + (rho^(k - 1) + 1) S3 + S4) +
    (rho^k (rho + 1)) (G4) (S1 + (rho^(k - 1) + 1) S3 + S4) -
    (G1 + (rho^k) G2) (S2) -
    rho^2 (G1 + (rho^(k - 1) + 1) G2) (S5) -
    rho^2 (G3 + (rho^k) G4) (S1 + (rho^(k - 1) + 1) S3) -
    rho^2 (G3 + (rho^k) G4) (S5) -
    (rho^(k - 1) + 1) (G2) (S1) -
    (1 + 1 / rho^k) (rho^(k - 1) + 1) (G1 + (rho^(k - 1) + 1) G2) (S3) -
    rho (G4) ((rho^(k - 1) + 1) S3 + S4) -
    rho^k (rho^(k - 1) + 1) (G2) (S3) +
    (rho - 1 / rho^k) (rho^(k - 1) + 1) (G3) (S3) +
    rho^k (G2) (S2 - S1) -
    rho (G3) (S4) -
    (rho^k) (G2) (S4);

(* Right hand side of identity in Proposition B.9 *)
RHSgs = -(G1 + (rho^(k - 1) + 1) G2 + G3 + (rho^(k + 1)) G4)
    (S1 + (1 + 1 / rho^k) (rho^(k - 1) + 1) S3 - (rho - 1 - 1 / rho^k) S4 + (rho) S5);

(* Verifying identity *)
LHSgs - RHSgs // FullSimplify
```

Out[ ]=
```
0
```

Verification of identity: certificate of (s, s) (Proposition B.10)

In[•]:=
```
(* Left hand side of identity in Proposition B.10 *)
LHSss = S1 (S2 - S1) +
    (S1 + (rho^(k - 1) + 1) S3 + S4) ((rho - 1 - 1 / rho^k) S4 + (rho + 1) S5) -
    (1 / 2) (S2)^2 -
    rho^2 (S1 + (rho^(k - 1) + 1) S3) (S5) -
    (rho^2 / 2) (S5)^2 -
    (1 + 1 / rho^k) (rho^(k - 1) + 1) (S3) (S1 + (rho^(k - 1) + 1) S3) -
    rho^k (rho^(k - 1) + 1) (S3)^2 +
    (rho - 1 / rho^k) (rho^(k - 1) + 1) (S3) (S4) +
    rho^k (S3) (S2 - S1) -
    rho (S4)^2 -
    rho^k (S3) (S4) +
    (1 / 2) (S2 - S1)^2 +
    (rho^2 / 2) (S5 - S4)^2 +
    (1 / 2) (rho^(k - 1) + 1) (rho^(k + 1) + 1) (S3)^2 -
    rho^k (S3) (S2 - S1) -
    rho (S3) (S4) -
    (1 / 2) (((rho^(k - 1) + 1) / rho^k) S3 - (rho - 1 / rho^k) S4 + (rho) S5)^2;

(* Right hand side of identity in Proposition B.10 *)
RHSss =
  - (1 / 2) (S1 + (1 + 1 / rho^k) (rho^(k - 1) + 1) S3 - (rho - 1 - 1 / rho^k) S4 + (rho) S5)^2;

(* Verifying identity *)
LHSss - RHSss // FullSimplify
```

Out[•]=
```
0
```

In[•]:=
```
CloudPublish[EvaluationNotebook[], "ProxVerification", Permissions → "Public"]
```

Out[•]=
```
CloudObject[https://www.wolframcloud.com/obj/alts0/MyNotebook]
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